Hong Kong Mathematics Olympiad (2005 – 2006)
Final Event 1 (Individual)
香港数学竞赛 (2005 – 2006)
决赛项目 1 (个人)

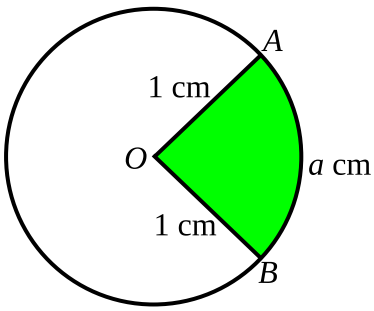
除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若 a 为实数且满足方程 $\log_2(x+3)-\log_2(x+1)=1$，求 a 的值。

If a is a real number satisfying the equation $\log_2(x+3)-\log_2(x+1)=1$, find the value of a.

2. 如图一，O 是半径为 1 cm 的圆的圆心。若弧 AB 的长度是 a cm 及着色部分扇形 OAB 的面积是 b cm^2，求 b 的值。(取 $\pi=3$)

In Figure 1, O is the center of the circle with radius 1 cm. If the length of the arc AB is equal to a cm and the area of the shaded sector OAB is equal to b cm^2, find the value of b. (Take $\pi=3$)



3. 一个正 C 边形的一只内角是$288b^\circ$，求 C 的值。

An interior angle of a regular C-sided polygon is $288b^\circ$, find the value of C.

4. 已知 C 是方程 $kx^2+2x+5=0$ 的一个根，其中 k 为常数。若 D 是另一个根，求 D 的值。

Given that C is a root of the equation $kx^2+2x+5=0$, where k is a constant. If D is another root, find the value of D.

Hong Kong Mathematics Olympiad (2005 – 2006)

Final Event 2 (Individual)

香港数学竞赛 (2005 – 2006)

决赛项目 2 (个人)

除非特别声明，答案须用数字表达，并化至最简。
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1. 已知 a 、 b 和 c 三数均不为 0 且 $a:b:c=6:3:1$。若 $R=\frac{3b^2}{2a^2+bc}$，求 R 的值。

Given that a , b and c are three numbers not equal to 0 and $a:b:c=6:3:1$. If $R=\frac{3b^2}{2a^2+bc}$, find the value of R .

2. 已知 $\frac{|k+R|}{|R|}=0$。若 $S=\frac{|k+2R|}{|2k+R|}$，求 S 的值。

Given $\frac{|k+R|}{|R|}=0$. If $S=\frac{|k+2R|}{|2k+R|}$, find the value of S .

3. 已知 $T=\sin 50^\circ\times\left(S+\sqrt{3}\times\tan 10^\circ\right)$，求 T 的值。

Given that $T=\sin 50^\circ\times\left(S+\sqrt{3}\times\tan 10^\circ\right)$, find the value of T .

4. 已知 x_0 和 y_0 是实数且满足方程组 $\begin{cases} y = \dfrac{T}{x} \\ y = |x| + T \end{cases}$ 。若 $W = x_0 + y_0$ ，求 W 的值。

Given that x_0 and y_0 are real numbers satisfying the system of equations $\begin{cases} y = \dfrac{T}{x} \\ y = |x| + T \end{cases}$. If $W = x_0 + y_0$, find the value of W.

Hong Kong Mathematics Olympiad (2005 – 2006)

Final Event 3 (Individual)

香港数学竞赛 (2005 – 2006)

决赛项目 3 (个人)

除非特别声明，答案须用数字表达，并化至最简。
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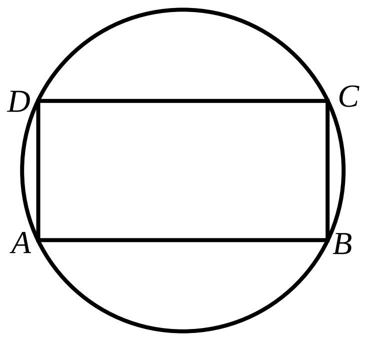
1. 已知 $\frac{2x-3}{x^2-x}=\frac{A}{x-1}+\frac{B}{x}$, 其中 A 和 B 是常数。若 $S=A^2+B^2$，求 S 的值。

Given that $\frac{2x-3}{x^2-x}=\frac{A}{x-1}+\frac{B}{x}$ where A and B are constants . If $S=A^2+B^2$, find the value of S .

2. 如图一，$ABCD$ 是圆内接长方形，$AB=(S-2)$ cm 及 $AD=(S-4)$ cm。若圆形的圆周是 R cm，求 R 的值。(取 $\pi=3$)

In Figure1, $ABCD$ is an inscribed rectangle , $AB=(S-2)$ cm and $AD=(S-4)$ cm . If the circumference of the circle is R cm , find the value of R . (take $\pi=3$)



图一

Figure 1

3. 已知整数 x 和满足 $\frac{R}{2}xy=21x+20y-13$ 。若 $T=xy$，求 T 的值。

Given that x and y are integers satisfying the equation $\frac{R}{2}xy=21x+20y-13$. If $T=xy$, find the value of T .

4. 设 a 是方程 $x^2-2x-T=0$ 的一个正根。若 $P=3+\cfrac{T}{2+\cfrac{T}{2+\cfrac{T}{2+\cfrac{T}{a}}}}$，求 P 值。

Let a be the positive root of the equation $x^2-2x-T=0$. If $P=3+\cfrac{T}{2+\cfrac{T}{2+\cfrac{T}{2+\cfrac{T}{a}}}}$, find the value of P.

Hong Kong Mathematics Olympiad (2005 – 2006)

Final Event 4 (Individual)

香港数学竞赛 (2005 – 2006)

决赛项目 4 (个人)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 设 $\frac{k}{4}=\left(1+\frac{1}{2}+\frac{1}{3}\right)\times\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)-\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)\times\left(\frac{1}{2}+\frac{1}{3}\right)$，求 k 的值。

   Let $\frac{k}{4}=\left(1+\frac{1}{2}+\frac{1}{3}\right)\times\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)-\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)\times\left(\frac{1}{2}+\frac{1}{3}\right)$, find the value of k.

2. 设 x 和 y 是实数且满足方程 $y^2+4y+4+\sqrt{x+y+k}=0$。若 $r=|xy|$，求 r 的值。

   Let x and y be real numbers satisfying the equation $y^2+4y+4+\sqrt{x+y+k}=0$. If $r=|xy|$, find the value of r.

3. 如图一，八个正数排成一列，从第二个数开始，每个数都等于前面两个数的乘积。已知第五个数是 $\frac{1}{r}$，而第八个数是 $\frac{1}{r^4}$。若第一个数是 s，求 s 的值。

   In Figure 1, there are eight positive numbers in series. Starting from the 3rd number, each number is the product of the previous two numbers. Given that the 5th number is $\frac{1}{r}$, and the 8th number is $\frac{1}{r^4}$. If the 1st number is s, find the value of s.

s				$\frac{1}{r}$			$\frac{1}{r^4}$

4. 设 $[x]$ 表示不大于 x 的最大整数，例如$[2.5] = 2$。若

$w=2+\left[10\times s^{2}\right]+\left[10\times s^{4}\right]+\left[10\times s^{6}\right]+\cdots+\left[10\times s^{2n}\right]+\cdots$，求 w 的值。

Let $[x]$ be the largest integer not greater than x, for example, $[2.5] = 2$. If

$w=2+\left[10\times s^{2}\right]+\left[10\times s^{4}\right]+\left[10\times s^{6}\right]+\cdots+\left[10\times s^{2n}\right]+\cdots$, find the value of w.